USN | | | | | | | | | | **10MAT41**

Fourth Semester B.E. Degree Examination, Jan./Feb. 2021
Engineering Mathematics – IV

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO full questions from each part.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Using the Taylor's series method, solve the initial value problem $\frac{dy}{dx} = x^2y - 1$, $y(0) = 1$ at the point $x = 0.1$. Consider terms upto third degree. **(06 Marks)**

b. Solve the differential equation $\frac{dy}{dx} = x - y^2$, under the initial condition $y(0) = 1$ by using the modified Euler's method, at the points $x = 0.1$ and $x = 0.2$. Take the step size $h = 0.1$ and carryout two modifications at each step. **(07 Marks)**

c. Using Adams-Bashforth method to solve the equation $\frac{dy}{dx} = \frac{x^2}{y^2 + 1}$ at $x = 1$ given the following table. Apply the corrector formula twice. **(07 Marks)**

x	0	0.25	0.50	0.75
y	1	1.0026	1.0206	1.0679

2 a. Use Picard's method to find y(0.1) and z(0.1) given that $\frac{dy}{dx} = x + z$; $\frac{dz}{dx} = x - y^2$ and $y(0) = 2$, $z(0) = 1$. Carry out two approximations. **(06 Marks)**

b. Using the Runge-Kutta method of fourth order, solve the differential equation $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} - 2xy = 1$ at $x = 0.1$ given $y(0) = 1$, $y'(0) = 0$. Take step length $h = 0.1$. Compute y(0.1). **(07 Marks)**

c. Using the Milne's method, obtain the solution of the equation $2\frac{d^2y}{dx^2} = 4x + \frac{dy}{dx}$ by computing the value of the dependent variable corresponding to the value 1.4 of the independent variable by the following data:

x	1	1.1	1.2	1.3
y	2	2.2156	2.4649	2.7514
y′	2	2.3178	2.6725	3.0657

(07 Marks)

3 a. Show that $w = z + e^z$ is analytic and hence find $\frac{dw}{dz}$. **(06 Marks)**

b. If $f(z) = u + iv$ is analytic find f(z) if $u - v = (x - y)(x^2 + 4xy + y^2)$. **(07 Marks)**

c. If the potential function is $\phi = 3x^2y - y^3$. Find the stream function. **(07 Marks)**

4 a. Find the bilinear transformation which maps the points $z_1 = -1$, $z_2 = 0$, $z_3 = 1$ on to the point $w_1 = 0$, $w_2 = i$, $w_3 = 3i$. **(06 Marks)**

b. Discuss the transformation $w = z + \frac{a^2}{z}$ **(07 Marks)**

c. Evaluate $\int_c \frac{e^{2z}}{(z+1)(z-2)}dz$ where c is the circle $|z| = 3$, using Cauchy's integral formula. **(07 Marks)**

PART – B

5 a. $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\cos x$ (06 Marks)

b. State and prove Rodrigue's formula for Legendre's polynomials. (07 Marks)

c. Express $f(x) = x^3 + 2x^2 - 4x + 5$ interms of Legendre's polynomials. (07 Marks)

6 a. A bag contains 4 white and 2 Red balls. Another bag contains 3 white and 5 Red balls. If a ball is drawn from each, find the probability that i) both are white ii) both are red iii) one is red and another is white. (06 Marks)

b. If A and B are events with $P(A) = 1/2$, $P(A \cup B) = 3/4$ and $P(\bar{B}) = 5/8$.
Find: i) $P(A \cap B)$ ii) $P(\bar{A} \cap \bar{B})$ iii) $P(\bar{A} \cup \bar{B})$ iv) $P(\bar{A} \cap B)$ (07 Marks)

c. A bag contains three coins, one of which is two headed and the other two are normal and fair. A coin is chosen at random from the bag and tossed four times in succession. If head turns up each time, what is the probability that this is the two header coin? (07 Marks)

7 a. The probability distribution of a finite random variable X is given by the following table:

x	-3	-2	-1	0	1	2	3
p(x)	k	2k	3k	4k	3k	2k	k

Determine the value of K and the mean, variance and standard deviation. Also find $p(x > 1)$ and $p(-1 < x \le 2)$. (06 Marks)

b. 2% of the fuses manufactured by a firm are found to be defective. Find the probability that a box containing 200 fuses contains
i) No defective fuses ii) 3 or more defective fuses. (07 Marks)

c. If x is an exponential variate with mean 5, evaluate i) $P(0 < x < 1)$ ii) $p(-\infty < x < 10)$ iii) $p(x \le 0 \text{ or } x \ge 1)$. (07 Marks)

8 a. A coin is tossed 1000 times and head turns up 540 times. Decide on the hypothesis that the coin is unbiased. (06 Marks)

b. Ten individuals are chosen at random from a population and their heights in inches are found to be 63, 63, 66, 67, 68, 69, 70, 70, 71, 71. Test the hypothesis that the mean height of the universe is 66 inches ($t_{.05} = 2.262$ for 9d.f). (07 Marks)

c. A sample analysis of examination results of 500 students was made. It was found that 220 students had failed, 170 had secured third class, 90 had secured second class and 20 had secured first class. Do these figures support the general examination result which is in the ratio 4:3:2:1 for the respective categories $(\chi^2_{0.05} = 7.81 \text{ for } 3d.f)$ (07 Marks)

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